

MIDTERM EXAMINATION  
MTH101- Calculus And Analytical Geometry

Question No: 1 ( Marks: 1 ) - Please choose one

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If  $f$  is a twice differentiable function at a stationary point  $x_0$  and  $f''(x_0) < 0$  then  $f$  has relative ..... At  $x_0$

- ▶ Minima
- ▶ **Maxima**
- ▶ None of these

Note:Maxima(If Maxima refers to local maximum)

Question No: 2 ( Marks: 1 ) - Please choose one

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A line  $x = x_0$  is called ----- for the graph of a function  $f$  if  $f(x) \rightarrow +\infty$  or  $f(x) \rightarrow -\infty$  as  $x$  approaches  $x_0$  from the right or from the left

- ▶ Horizontal asymptotes
- ▶ None of these
- ▶ **Vertical asymptotes**

Question No: 3 ( Marks: 1 ) - Please choose one

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A line  $y = y_0$  is called a ..... for the graph  $f$  if  $\lim_{x \rightarrow +\infty} f(x) = y_0$  or  $\lim_{x \rightarrow -\infty} f(x) = y_0$

- ▶ Vertical asymptotes
- ▶ Horizontal asymptotes
- ▶ **None of these**

Question No: 4 ( Marks: 1 ) - Please choose one

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According to Power-Rule of differentiation, if  $f(x) = x^n$  where  $n$  is a real number, then

$$\frac{d}{dx}[x^n] =$$

- ▶  $x^{n-1}$
- ▶  **$n x^{n-1}$**

- ▶  $n x^{n+1}$
- ▶  $(n-1)x^{n+1}$

**Question No: 5 ( Marks: 1 ) - Please choose one**

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$$y = \frac{1}{1-x} \quad \frac{dy}{dx} =$$

If \_\_\_\_\_ then

- ▶ 1
- ▶ -1
- ▶  $\frac{1}{(1-x)^2}$
- ▶  $\frac{-1}{(1-x)^2}$

**Question No: 6 ( Marks: 1 ) - Please choose one**

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$$\text{If } xy = 4 \text{ then } \frac{dy}{dx} =$$

- ▶ 0
- ▶  $\frac{-1}{x^2}$
- ▶  $\frac{4}{x^2}$
- ▶  $\frac{-4}{x^2}$

**Question No: 7 ( Marks: 1 ) - Please choose one**

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$$\text{If } 2x - y = -3 \text{ then } \frac{dy}{dx} =$$

- ▶ 2
- ▶ -2
- ▶ 0
- ▶ -3

**Question No: 8 ( Marks: 1 ) - Please choose one**

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$$\frac{d}{dx}[\sec x] = \underline{\hspace{2cm}}$$

- ☐  $\frac{1}{1 + \sin^2 x}$
- ☐  $\frac{-\sin x}{1 + \sin^2 x}$
- ☐  $\frac{1}{1 - \sin^2 x}$
- ☒  $\frac{\sin x}{1 - \sin^2 x}$

**Question No: 9 ( Marks: 1 ) - Please choose one**

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$$30^0 = \underline{\hspace{2cm}}$$

- ☐  $\frac{\pi}{3}$
- ☐  $\frac{\pi}{4}$
- ☒  $\frac{\pi}{6}$
- ☐  $\frac{\pi}{2}$

**Question No: 10 ( Marks: 1 ) - Please choose one**

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Consider a function  $h(x)$  and a constant  $c$  then

$$\frac{d}{dx}((c) \{h(x)\}) = \underline{\hspace{2cm}}$$

- ☐ 0
- ☐  $\frac{d}{dx}(h(x))$

- $\frac{d}{dx}(h(cx))$
- $c \frac{d}{dx}(h(x))$

**Question No: 11 ( Marks: 1 ) - Please choose one**

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$$\frac{d}{dx}[\operatorname{cosec} x] = \underline{\hspace{2cm}}$$

- $\frac{1}{1 + \cos^2 x}$
- $\frac{-\cos x}{1 - \cos^2 x}$
- $\frac{-\cos x}{1 - \cos^2 x}$
- $\frac{1}{1 - \cos^2 x}$

**Question No: 12 ( Marks: 1 ) - Please choose one**

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Chain rule is a rule for differentiating \_\_\_\_\_ of functions.

- **Product**
- Sum
- Difference
- Composition

**Question No: 13 ( Marks: 1 ) - Please choose one**

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$$\frac{d}{dx}[x^n] = nx^{n-1}$$

The power rule, \_\_\_\_\_ holds if n is \_\_\_\_\_

- **An integer**
- A rational number
- An irrational number
- All of the above

**Question No: 14 ( Marks: 1 ) - Please choose one**

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Let a function  $f$  be defined on an interval, and let  $x_1$  and  $x_2$  denote points in that

interval. If  $f(x_1) < f(x_2)$  whenever  $x_1 < x_2$  then which of the following statement is correct?

- ▶  $f$  is an increasing function.
- ▶  $f$  is a decreasing function.
- ▶  $f$  is a constant function.

**Question No: 15 ( Marks: 1 ) - Please choose one**

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If  $f''(x) > 0$  on an open interval (a,b), then which of the following statement is correct?

- ▶  $f$  is concave up on (a, b).
- ▶  $f$  is concave down on (a, b).
- ▶  $f$  is linear on (a, b).

**Question No: 16 ( Marks: 1 ) - Please choose one**

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If  $f''(x) < 0$  on an open interval (a,b) then which of the following statement is correct?

- ▶  $f$  is concave up on (a, b).
- ▶  $f$  is concave down on (a, b).
- ▶  $f$  is linear on (a, b).

**Question No: 17 ( Marks: 1 ) - Please choose one**

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If  $x > 0$  then  $\frac{d}{dx}[\ln x] =$  \_\_\_\_\_

- ▶ 1
- ▶  $x$
- ▶  $\frac{1}{x}$
- ▶  $\ln \frac{1}{x}$

**Question No: 18 ( Marks: 1 ) - Please choose one**

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If  $b > 0$  then  $\frac{d}{dx}[b^x] =$  \_\_\_\_\_

- ▶ 0
- ▶  $xb^{x-1}$
- ▶  $\ln b$
- ▶  $b^x \ln b$

**Question No: 19 ( Marks: 1 ) - Please choose one**

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Let  $y = (x^3 + 2x)^{37}$ . Which of the following is correct?

☐  $\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$



$\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$



$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$



$\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$



**Question No: 20 ( Marks: 1 ) - Please choose one**

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What is the base of natural logarithm?

☒ 2.71

☐ 10

☐ 5

☐ Any real number

**Question No: 21 ( Marks: 1 ) - Please choose one**

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Let  $x_0$  be critical points of the function  $f$ . Those critical points for which  $f'(x_0) = 0$  are called \_\_\_\_\_ of  $f$

☐ Local points

☐ End points

☒ Stationary points

**Question No: 22 ( Marks: 1 ) - Please choose one**

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$\log_b a^r = \underline{\hspace{2cm}}$

☒  $a \log_b r$

☐  $r \log_b a$

$\frac{\log_b a}{\log_b r}$



☐  $\log_b a + \log_b r$

**Question No: 23 ( Marks: 1 ) - Please choose one**

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$\log_b \frac{1}{c} = \underline{\hspace{2cm}}$

☐  $\log_b c$

☐  $1 - \log_b c$



☒  $-\log_b c$

☐  $1 + \log_b c$

**Question No: 24 ( Marks: 1 ) - Please choose one**

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$\log_b \frac{1}{t} = \underline{\hspace{2cm}}$

☐  $\log_b t$

☐  $1 - \log_b t$



☐  $1 + \log_b t$



☒  $-\log_b t$

**Question No: 25 ( Marks: 3 )**

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If  $f(x) = x^4 - 8x^2$ , determine all relative extrema for the function. Using First Derivative Test.

**Solution:**

$$f = x^4 - 8x^2$$

$$f' = 4x^3 - 16x^1$$

$$f' = 0$$

$$4x^3 - 16x = 0$$

$$x(4x^2 - 16) = 0$$

$$x = 0$$

$$4x^2 - 16 = 0$$

$$x^2 = \frac{16}{4}$$

$$x^2 = 4$$

$$x = \pm 2$$

Relative extrema (0, ±2)....Ans

**Question No: 26 ( Marks: 5 )**

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Differentiate  $y = x^{-2}(4 + 3x^{-3})$

**Solution:**

$$y = 4x^{-2} + 3x^{-3} \cdot x^{-2}$$

$$= 4x^{-2} + 3x^{-5}$$

$$\frac{dy}{dx} = 4 \frac{d}{dx}(x^{-2}) + 3 \frac{d}{dx}(x^{-5})$$

$$= 4(-2)x^{-2-1} + 3 \frac{d}{dx} - 5x^{-5-1}$$

$$= -8x^{-3} + (-15x^{-6})$$

$$\frac{dy}{dx} = -8x^{-3} - 15x^{-6} \dots \text{Ans}$$

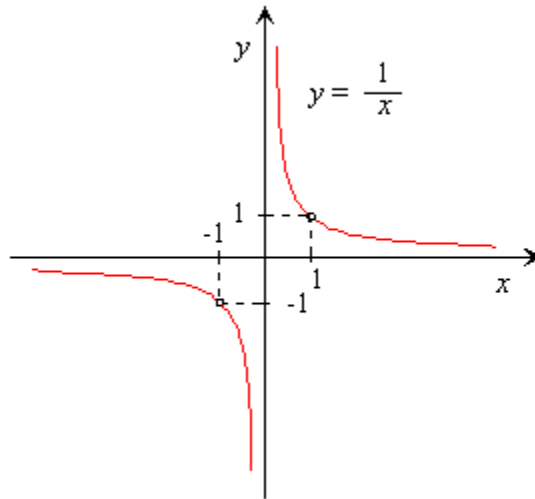
**Question No: 27 ( Marks: 10 )**

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$$f(x) = \frac{1}{x}$$

Determine the intervals in which the graph of the function is concave upward or downward.





**Solution:**

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

X	1	2	3	4
F(x)	-1	-0.25	-0.11	-0.625

Conclusion:

$f'(x)$  is increasing when x is from  $(0, \infty)$

so,

It is concave up.

$f'(x)$  is decreasing when x is from  $(-\infty, 0)$

so,

It is concave down